4. Force

- **Inertia**

    - The property of a body to resist any change is called inertia e. g. when a horse starts suddenly, the rider falls backwards due to the inertia of rest of the upper part of his body.

- **Newton's First Law**

    - Every body continues to be in its state of rest or uniform motion in a straight line, unless compelled by some external force acting on it.
    - Example: Any object continues lying where it is, unless it is moved.
    - Newton's first law gives the definition of Inertia.

Momentum:

- Momentum of a body is the product of its mass, *m* and velocity, *v*, and is denoted by *P*.
- It is a vector quantity.
- SI unit → kg ms^{-1}

Newton's Second Law of Motion

- The rate of change of linear momentum of a body is directly proportional to the external force applied on the body, and this change takes place always in the direction of force applied.

  i.e. ΔpΔtα Fext

- Second law is the real law of motion.

Impulse

- Impulse of a force is a measure of the total effect of the force.

  Impulse = Force × Time

- SI unit of impulse are N-s and kg-ms^{-1}
- Forces which act on bodies for a short time are called impulsive forces.

- **Third Law of motion**
  - **Statement:** To every action, there is always an equal and opposite reaction.

- **Some Important Points about the Third Law**
  - Forces always occur in pairs. Force exerted on a body A by a body B is equal and opposite to the force exerted on body B by body A.
  - There is no cause-effect relationship implied in the third law. The force on body A by body B, and the force on body B by body A act at the same instant.
  - Forces of action and reaction act always on different bodies. Hence, they never cancel each other.

Frame of reference:

- To locate the physical quantities like position, velocity and acceleration of a particle, a frame of reference is needed. Generally, a set of three mutually perpendicular axes is taken as the frame of reference .

There are two types of frame of reference:

(I) Inertial frame of reference:

- A frame of reference moving with constant velocity or at rest with respect to the ground is called inertial frame of reference. In this frame Newton's laws of motion are valid. For example, the Earth.

(II) Non-inertial frame of reference:

- A frame of reference which is moving with an acceleration is called a non-inertial frame of reference. Newton's laws of motion are not valid in a non-inertial frame of reference. For example, a rotating frame.

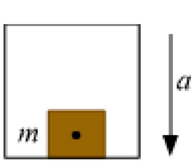
Pseudo Force:

- Pseudo forces arise when Newton's laws of motion are applied in a non-inertial (accelerating) frame of reference.
- These forces have no real existence but must be taken into account in an accelerating frame of reference to make Newton's laws of motion applicable to the system.
- Centrifugal force is the pseudo force for a body in circular motion. The real force in this case is centripetal force that is directed inwards.

Real forces:

- Real forces arise due to interaction between objects.
- These forces have specific source and origin outside the body experiencing the force.
- These forces are explained on the basis of fundamental interactions.

For a block of mass m placed in an elevator moving down with constant acceleration a:



Case (I)

For an inertial frame of reference (ground frame),

$N = m\,(g - a)$ …(I)

Case (II)

For non-inertial frame of reference (i.e. from the lift),

$N = m\,(g - a)$

- **Momentum:** Momentum of a body is the product of its mass, *m* and velocity, *v* and is denoted by *P*.

  $$\vec{P} = m\vec{v}$$

- **Conservation of momentum:** In an isolated system, the vector sum of the linear momenta of all the bodies of the system is conserved and is not affected due to their mutual action and reaction. That is,

  Initial momentum of the system = Final momentum of the system

- **Examples of conservation of momentum**

  1. Recoil of a gun on firing a bullet
  2. Backward movement of a boat when a person jumps out of it
  3. Upward motion of a rocket due to downward motion of exhaust gases

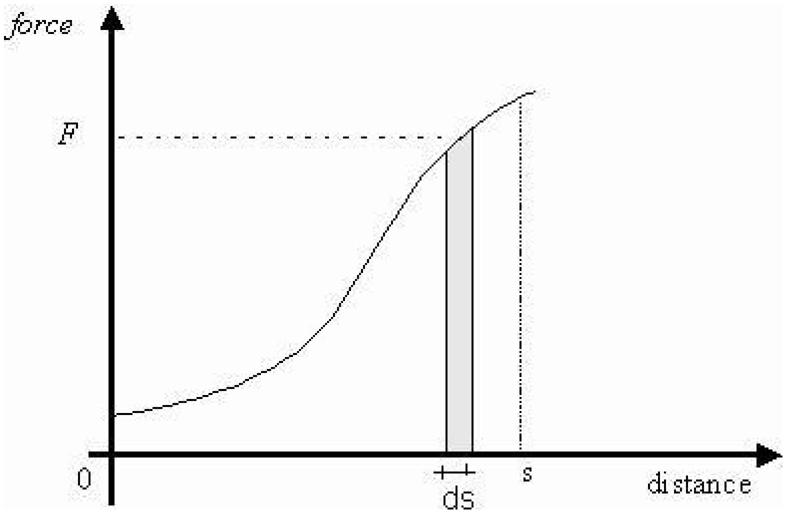
Work – energy theorem

- Work done by net force is equal to the change in kinetic energy of the body
- $W = \vec{F}.\vec{d} = Fd\ \cos\theta$ (Dot product, hence it is a scalar quantity)
- No work is done if
  - displacement is zero
  - force is zero
  - force and displacement are mutually perpendicular i.e.,

    $$\theta = \frac{\pi}{2} = 90^\circ$$

- Kinetic energy, $K = \frac{1}{2}m\vec{v}.\vec{v} = \frac{1}{2}mv^2$

- **Work done by variable force**



- **Work done is the area subtended by the curve on the distance axis.**

$$W = \int_{x_i}^{x_f} F(x)\mathrm{d}x$$

- **Work – energy theorem for variable force**

$$\mathrm{d}K = F\mathrm{d}x$$

$$K_f - K_i = \int_{x_i}^{x_f} F\mathrm{d}x$$

Types of Collision

- Elastic collision – Those collisions in which both momentum and kinetic energy of the system are conserved.
- Inelastic collision – Those collisions in which momentum of the system is conserved, but kinetic energy is not conserved.

Characteristics of elastic collision

- total energy of the syatem is conserved
- linear momentum is conserved
- kinetic energy is conserved

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- A couple of forces are needed to rotate a body about an axis.
- Torque represents the turning force acting on an object. It can be either clockwise or anticlockwise, depending upon how the force is applied.

Torque (τ) = Force (F) × Perpendicular distance (D)

- Torque is also known as moment of force.
- A clockwise torque tends to turn an object in the clockwise direction. Similarly, an anticlockwise torque tends to turn an object in the anticlockwise direction.
- The unit of torque is Nm (newton-metre).
- **Couple:** When two equal and opposite forces act on a body and their lines of action do not coincide, then the body will not move but rotate about the given axis.

  - **Moment of force (torque)**

$$\bar{\tau} = \bar{r} \times \bar{F}$$
$$|\bar{\tau}| = rF\sin\theta = rF_{\perp} = r_{\perp}F \quad (\text{unit} \rightarrow \text{kg m}^2\ \text{s}^{-2})$$

- **Angular momentum**

$$\bar{l} = \bar{r} \times \bar{p} \quad (\bar{p} = \text{linear momentum})$$

$$\frac{d\bar{l}}{dt} = \bar{\tau}_{ext}$$

  - For a rigid body, $\bar{L} = \sum \bar{l}_i = \sum \bar{r}_i \times \bar{p}_i$ and $\frac{d\bar{L}}{dt} = \sum \bar{\tau}_i$

$$\bar{\tau} = \bar{\tau}_{ext} + \bar{\tau}_{int}$$

  - Total angular momentum is conserved if total external torque is zero on a system.

- Centre of mass of a body is a point at which the whole mass of the body is supposes to be concentrated.
- Position of centre of mass of discreet distribution of mass

$$X = \frac{m_1x_1 + m_2x_2 + \ldots + m_nx_n}{m_1 + m_2 + \ldots + m_n} = \frac{\sum m_ix_i}{\sum m_i}$$
$$Y = \frac{m_1y_1 + m_2y_2 + \ldots + m_ny_n}{m_1 + m_2 + \ldots + m_n} = \frac{\sum m_iy_i}{\sum m_i}$$

- Position of centre of mass of continuous distribution of mass

$$\vec{R} = \frac{1}{M}\int \vec{r}\, dm$$

1. Centre of gravity of an object is a point where all the weight of the body is supposed to be concentrated.

2. Any object can be balanced about this point by applying an opposite force equal to the weight of the body, along the vertical line passing through the centre of gravity.

3. This point can lie either inside or outside the body depending upon its shape.

4. A body can be supported at the point which may not be the center of gravity of the body. However, in this case the body will be less stable.

5. When the net force acting on an object is zero, the object is said to be in a state of equilibrium.

6. On the basis of stability, there are three types of equilibriums; stable equilibrium, unstable equilibrium and neutral equilibrium.

7. If an object continues to be in the state of equilibrium, even after being applied with some small disturbance, then the object is said to be in **stable equilibrium**.

8. If an object changes its position permanently, after being applied with some small disturbance, then the object is said to be in **unstable equilibrium**.

9. On being disturbed, if a body always gains a similar position in such way that its potential energy does not change, then the body is said to be in **neutral equilibrium**.

Types of motion

- **Translational Motion**
  - Motion of a rigid body along a staright line path.
  - All the particles of the body move together i.e, they have the same velocity at any instant of time.

- **Rotational Motion**
  - Motion of rigid body about a fix point.
  - Every particle of the body moves in concentric circles about the fix point.

- **Combination of translational and rotational motion**
  - Motion of body in which body rotates while moving.
  - Motion of a tyer on the road.

- Mechanical equilibrium
  - In mechanical equilibrium, total external force is zero and total external torque is zero.

- Translational equilibrium
  - When a body is in translational equilibrium, it will be either at rest ($v = 0$) or in uniform motion.
  - The body will have zero linear acceleration.

- In equilibrium, potential energy of the body is constant (maximum or minimum).
- Rotational equilibrium
  - A body is in rotational equilibrium, when algebraic sum of moments of all the forces acting on the body about a fixed point is zero.
  - Angular acceleration of the body in rotational equilibrium will be zero.
- At centre of gravity, total gravitational torque is zero.